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9231/22

October/November 2023

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

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- 1 Find the Maclaurin's series for $\ln(x+2) + \ln(x^2+5)$ up to and including the term in x^2 . [5]

[illegible]

2 It is given that

$$x = 1 + \frac{1}{t} \quad \text{and} \quad y = te^t.$$

- (a) Show that $\frac{dy}{dx} = -e^t(t^3 + t^2)$. [3]

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- (b) Find $\frac{d^2y}{dx^2}$ in terms of t . [4]

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- 3 (a) Use de Moivre's theorem to show that

$$\cos 5\theta = 16 \cos^5 \theta - 20 \cos^3 \theta + 5 \cos \theta. \quad [4]$$

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- (b) Hence obtain the roots of the equation

$$32x^5 - 40x^3 + 10x - \sqrt{2} = 0$$

in the form $\cos(q\pi)$, where q is a rational number. [4]

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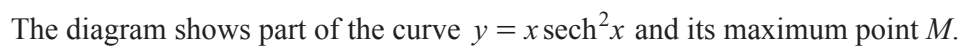
4 Find the solution of the differential equation

$$\frac{dy}{dx} + 3y = \sin x$$

for which $y = 1$ when $x = 0$. Give your answer in the form $y = f(x)$.

[9]

[illegible]



- and verify that this equation has a root between 0.7 and 0.8.

[illegible]

(b) By considering a suitable set of rectangles, use the diagram to show that

$$\sum_{r=2}^n r \operatorname{sech}^2 r < n \tanh n + \ln \operatorname{sech} n - \tanh 1 - \ln \operatorname{sech} 1. \quad [6]$$

[illegible]

6 The matrix $\mathbf{P}$ is given by

$$\mathbf{P} = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{pmatrix}.$$

(a) State the eigenvalues of $\mathbf{P}$. [1]

(b) Use the characteristic equation of $\mathbf{P}$ to find $\mathbf{P}^{-1}$. [4]

The 3×3 matrix $\mathbf{A}$ has distinct non-zero eigenvalues $a, \frac{1}{2}, 2$ with corresponding eigenvectors

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix},$$

respectively.

- (c) Find $\mathbf{A}^{-1}$ in terms of a .

[5]

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- 7 (a) Starting from the definitions of cosh and sinh in terms of exponentials, prove that

$$2 \sinh^2 A = \cosh 2A - 1. \quad [3]$$

[illegible]

- (b)** A curve has equation $y = x^2$, for $0 \leq x \leq \frac{2}{3}$. The area of the surface generated when the curve is rotated through 2π radians about the x -axis is denoted by S .

Use the substitution $x = \frac{1}{2} \sinh u$ to show that $S = \frac{1}{32} \pi \left(\frac{820}{81} - \ln 3 \right)$. [9]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

8 It is given that $v = y^4$ and

$$y^3 \frac{d^2 y}{dx^2} + 3y^2 \left(\frac{dy}{dx} \right)^2 + y^3 \frac{dy}{dx} + y^4 = e^{-2x}.$$

(a) Show that

$$\frac{d^2v}{dx^2} + \frac{dv}{dx} + 4v = 4e^{-2x}. \quad [4]$$

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- (b)** Find y in terms of x , given that, when $x = 0$, $y = 1$ and $\frac{dy}{dx} = -\frac{3}{8}$. [10]

[illegible]

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